Section 3.1 Increasing and Decreasing functions and Relative Maxima and Minima

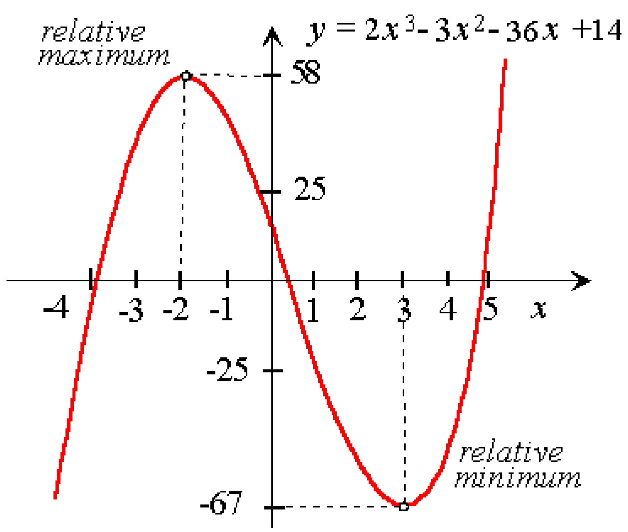
(Minimum Homework: 1 – 11 odds 13, 17, 21, 25)

A point is a **relative (local) *maximum point*** provided it is higher than any point close to it.

The **relative *(local) maximum value*** is the y-coordinate of the relative maximum point.

A point is a **relative (local) *minimum point*** provided it is lower than any point close to it.

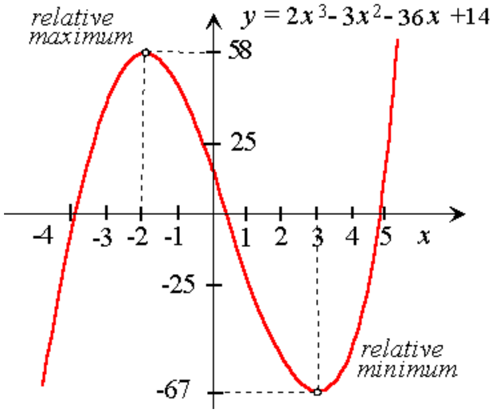
The ***relative (local) minimum value*** is the y-coordinate of the relative minimum point.



It would be proper to say the following with regards to the graph in the left panel.

- $(-2,58)$ is a relative maximum point
- The relative maximum value is $y = 58$ which occurs when $x = -2$
- $(3,-67)$ is a relative minimum point.
- The relative minimum value is $y = -67$ which occurs when $x = 3$.

We are going to need to be able to describe regions in graphs in this section.

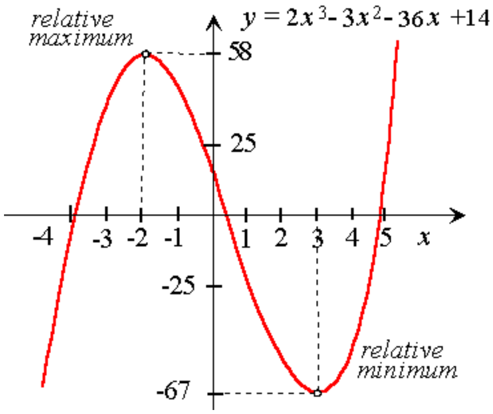


We need to be able to describe three regions in this graph using only round parenthesis and only x-values.
The regions we need to describe are:

- Left of relative maximum point
- Right of relative minimum point
- Between the relative maximum and minimum points.

Left of relative maximum point
Region starts at the far-left end of the x-axis $(x = -\infty)$
Region ends at the relative maximum point $(x = -2)$

Right of relative minimum point
Region starts at relative minimum point $(x = 3)$
Region ends at far-right end of x-axis $(x = \infty)$



Region between the relative maximum and relative minimum points.

Region starts at relative maximum point $(x = -2)$

Region ends at relative minimum point $(x = 3)$

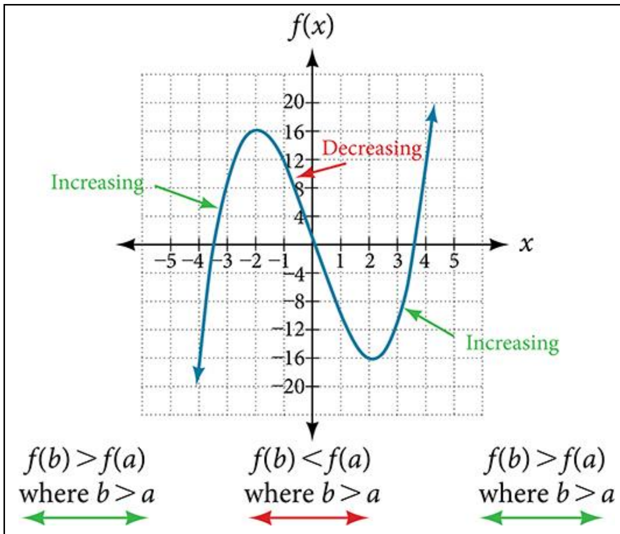
This is how the regions will need to be described in chapter 3. (only x-values and only round parenthesis)

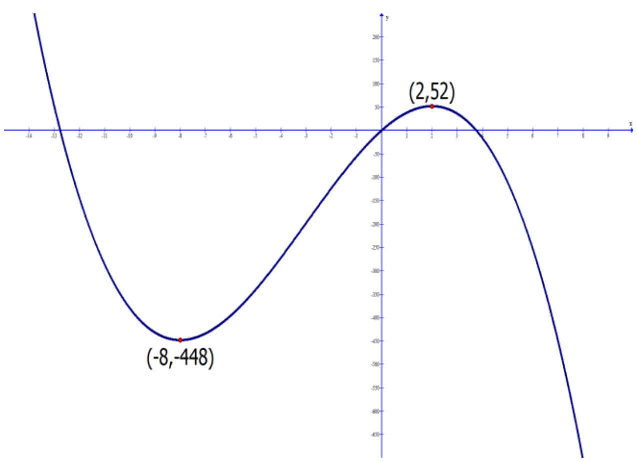
- Left of relative maximum point $(-\infty, -2)$
- Right of relative minimum point $(3, \infty)$
- Between the relative maximum and minimum points. $(-2,3)$

Using a Graph to Determine Where a Function is Increasing, Decreasing

A graph of a function is ***increasing*** over an open interval provided the y-coordinates of the points in the interval get larger, or equivalently the graph gets higher as it moves from left to right over the interval.

A graph of a function is ***decreasing*** over an open interval provided the $y - coordinates$ of the points in the interval get smaller, or equivalently the graph gets lower as it moves from left to right over the interval.

	This function has a relative maximum point of $(-2,16)$. It has a relative minimum point of $(2,-16)$ The graph of the function is increasing • left of relative maximum point $(-\infty,-2)$ • right of relative minimum point $(2,\infty)$ The graph of the function is decreasing • between the relative maximum and minimum points. $(-2,2)$

For example: Use the graph of f(x) to determine: a) interval(s) where the graph is increasing. b) interval(s) where the graph is decreasing. c) the coordinates of local maximum point, if any d) the relative maximum value e) the coordinates of the relative minimum point if any f) the relative minimum value  .	a) Increasing between the relative minimum point and the relative maximum point. $increasing\ (-8,2)$ b) Decreasing in two intervals. To left of relative minimum point: $(-\infty,-8)$ To right of relative maximum point: $(2,\infty)$ Decreasing: $(-\infty,-8) \cup (2,\infty)$ c) Relative maximum point $(2,52)$ d) Relative maximum value $y = 52$ which occurs when $x = 2$. e) Relative minimum point $(-8,-448)$ f) Relative minimum value: $y = \ -448$ which occurs when $x = -8$.

We need to use Calculus to find the intervals where a graph is increasing and decreasing and to find the maximum and minimum points.

Here are the steps to find the intervals where a graph is increasing and decreasing.

1) Find the derivative of the given function.
2) Find the critical numbers for the derivative.

- Any value of x that makes the derivative equal to zero.
- Any value of x that makes the derivative undefined.

3) Plot the critical numbers on a number line that also includes $-\infty \; and \; \infty$.
4) Create interval(s) using only round parenthesis.
5) Pick a number inside the interval and plug it into the derivative.
6) Determine whether the graph is increasing or decreasing in that interval.

- Graph is increasing when the result is positive.
- Graph is decreasing when the result is negative.

7) Place directional arrows in each interval to signify whether the graph in increasing or decreasing.

- Determine if each critical point is a relative maximum point, a relative minimum point, or neither.

8) Find the y-coordinates of any relative maximum or relative minimum points.

9) Write your answer.

Example: $f(x) = x^3 - 24x^2 + 18$

Find the following:

a) f'(x)
b) the critical numbers
c) interval(s) where the graph is increasing.
d) interval(s) where the graph is decreasing.
e) the coordinates of relative maximum point, if any
f) the relative maximum value
g) the coordinates of the relative minimum point if any
h) the relative minimum value

See next few pages for work / answers:

1) Find the derivative of the given function.

$$f'(x) = 3x^2 - 48x$$

2) Find the critical numbers for the derivative.

- Any value of x that makes the derivative equal to zero.
- Any value of x that makes the derivative undefined.

$3x^2 - 48x = 0$

$3x(x - 16) = 0$

$3x = 0$ $\qquad$ $x - 16 = 0$

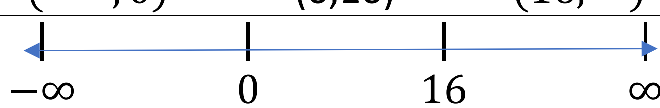
$x = 0$ $\qquad$ $x = 16$

Critical numbers $x = 0, 16$

3) Plot the critical numbers on a number line that also includes $-\infty$ *and* ∞.

4) Create interval(s) using only round parenthesis.

$(-\infty, 0)$ $\qquad$ (0,16) $\qquad$ $(16, \infty)$



5) Pick a number inside the interval and plug it into the derivative.

$(-\infty, 0)$ $choose\ x = -1$

$f'(-1) = 3(-1)^2 - 48(-1) = 51$

$(0,16)$ $choose\ x = 1$

$f'(1) = 3(1)^2 - 48(1) = -45$

$(16, \infty)$ $choose\ x = 17$

$f'(17) = 3(17)^2 - 48(17) = 51$

6) Determine whether the graph is increasing or decreasing in that interval.

- Graph is increasing when the result is positive.
- Graph is decreasing when the result is negative.

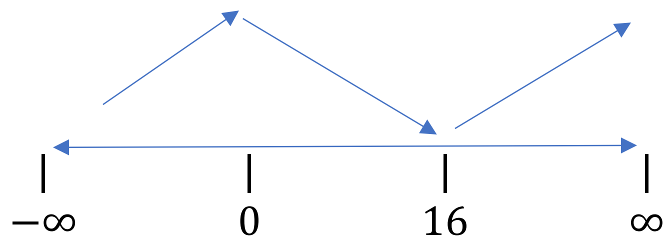
$(-\infty, 0)$ positive - increasing

$(0,16)$ negative - decreasing

$(16, \infty)$ positive - increasing

7) Place directional arrows in each interval to signify whether the graph in increasing or decreasing.

- Determine if each critical point is a relative maximum point, a relative minimum point, or neither.



Relative maximum point at $x = 0$

Relative minimum point at $x = 16$

8) Find the y-coordinates of any relative maximum or relative minimum points.

y-coordinate of relative maximum: $y = f(0) = (0)^3 - 24(0)^2 + 18 = 18$

relative maximum point $(0,18)$

y-coordinate of relative minimum: $y = f(16) = (16)^3 - 24(16)^2 + 18 = -2030$

relative minimum point $(16, -2030)$

9) Write your answer.

a) f'(x) $f'(x) = 3x^2 - 48x$

b) the critical numbers $x = 0{,}16$

c) interval(s) where the graph is increasing. $(-\infty, 0) \cup (16, \infty)$

d) interval(s) where the graph is decreasing. $(0{,}16)$

e) the coordinates of relative maximum point if any $(0, 18)$

f) the relative maximum value $relative\ maximum\ value\ y = 18\ which\ occurs\ when\ x = 0$

g) the coordinates of the relative minimum point if any $(16, -2030)$

h) the relative minimum value

$relative\ minimum\ value\ \ y = -2030\ which\ occurs\ when\ x = 16$

(Minimum Homework: 1 – 11 odds 13, 17, 23, 27)

1-12: Find:

a) interval(s) where the graph is increasing.

b) interval(s) where the graph is decreasing.

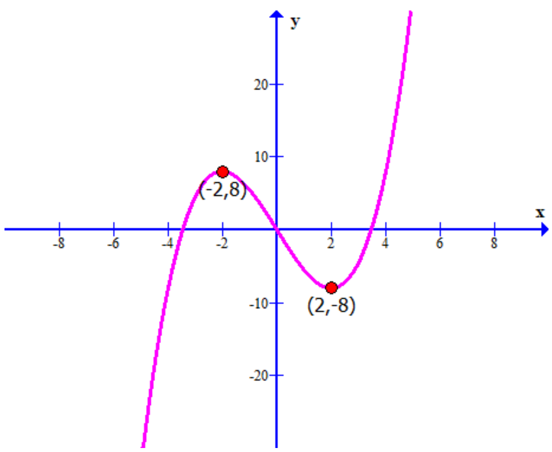
c) the coordinates of relative maximum point, if any

d) the relative maximum value

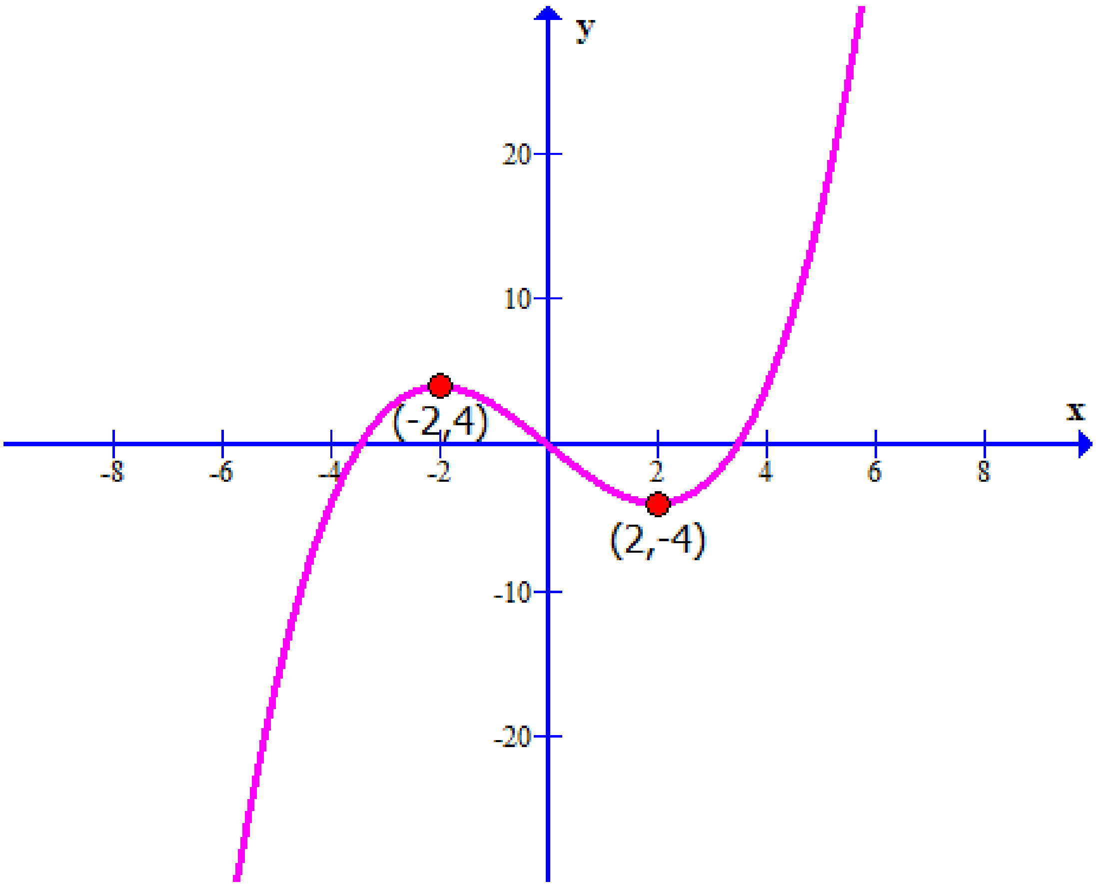
e) the coordinates of the relative minimum point if any

f) the relative minimum value

1)

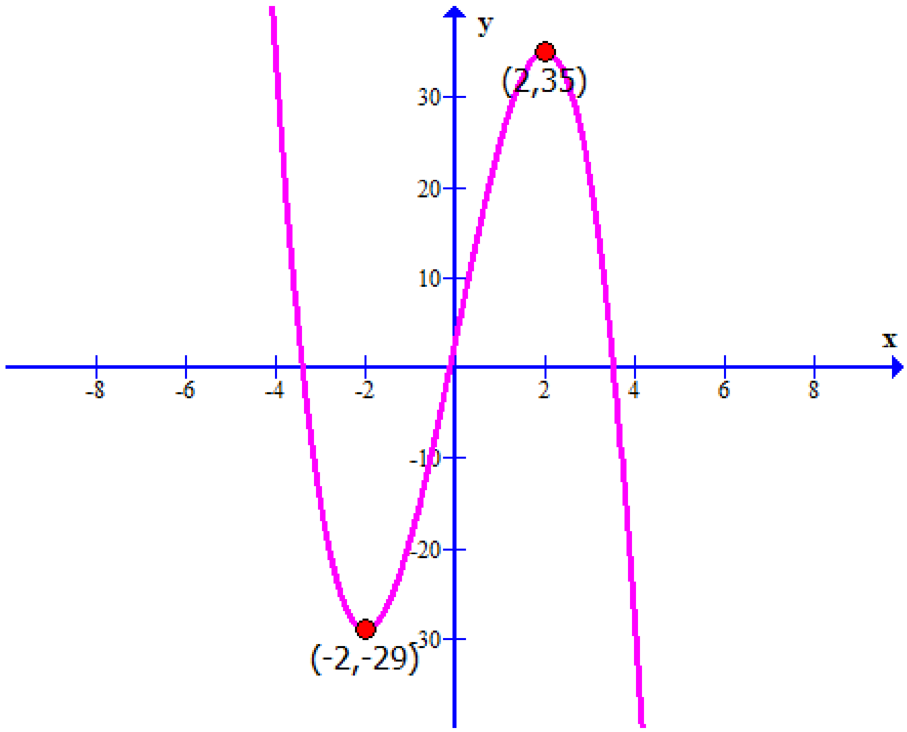


2)

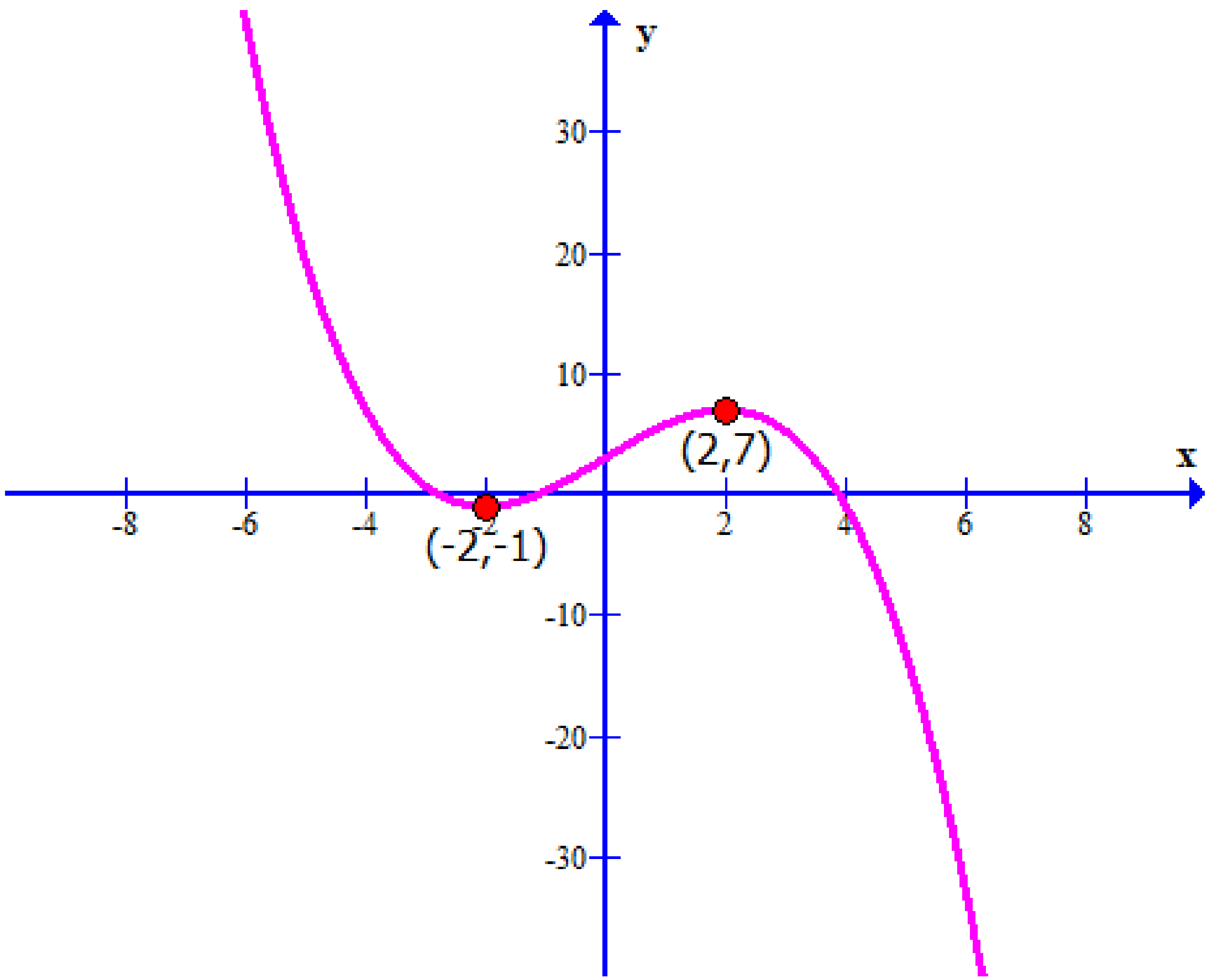


a) interval(s) where the graph is increasing. $(-\infty, -2) \cup (2, \infty)$
b) interval(s) where the graph is decreasing. $(-2,2)$
c) the coordinates of relative maximum point if any $(-2,4)$
d) the relative maximum value $y = 4\ when\ x = \ -2$
e) the coordinates of the relative minimum point if any $(2, -4)$
f) the relative minimum value $y = -4\ when\ x = 2$

3)



4)



a) interval(s) where the graph is increasing. $(-2,2)$

b) interval(s) where the graph is decreasing. $(-\infty,-2) \cup (2,\infty)$

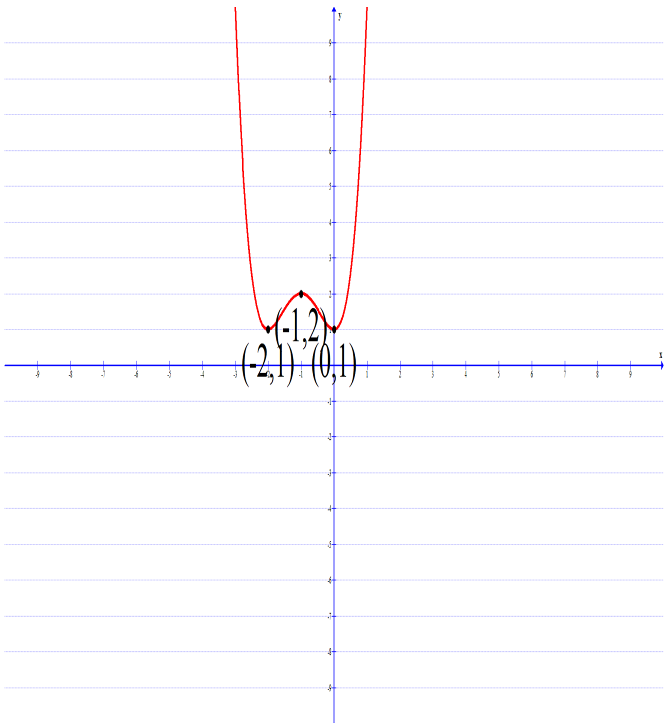
c) the coordinates of relative maximum point if any $(-2,7)$

d) the relative maximum value $y = 7 \; when \; x = 2$

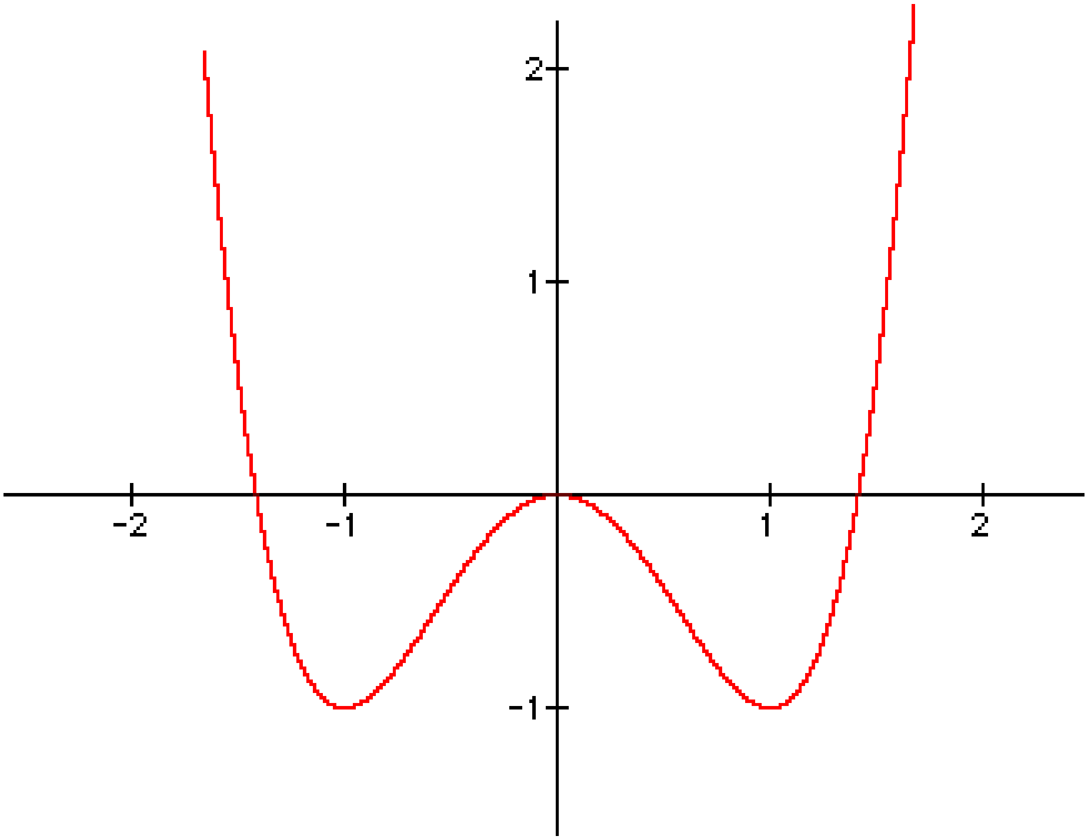
e) the coordinates of the relative minimum point if any $(-2,-1)$

f) the relative minimum value $y = -1 \; when \; x = -2$

5)



6)



a) interval(s) where the graph is increasing. $(-1,0) \cup (1,\infty)$

b) interval(s) where the graph is decreasing. $(-\infty,-1) \cup (0,1)$

c) the coordinates of relative maximum point if any $(0,0)$

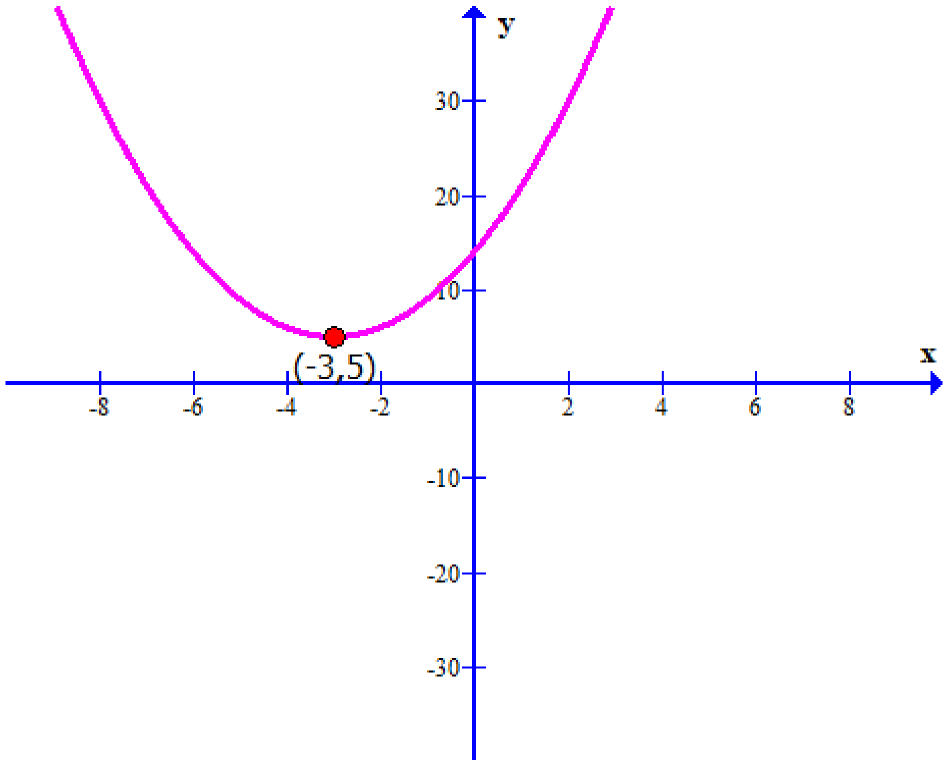
d) the relative maximum value $y = 0\ when\ x = 0$

e) the coordinates of the relative minimum point if any

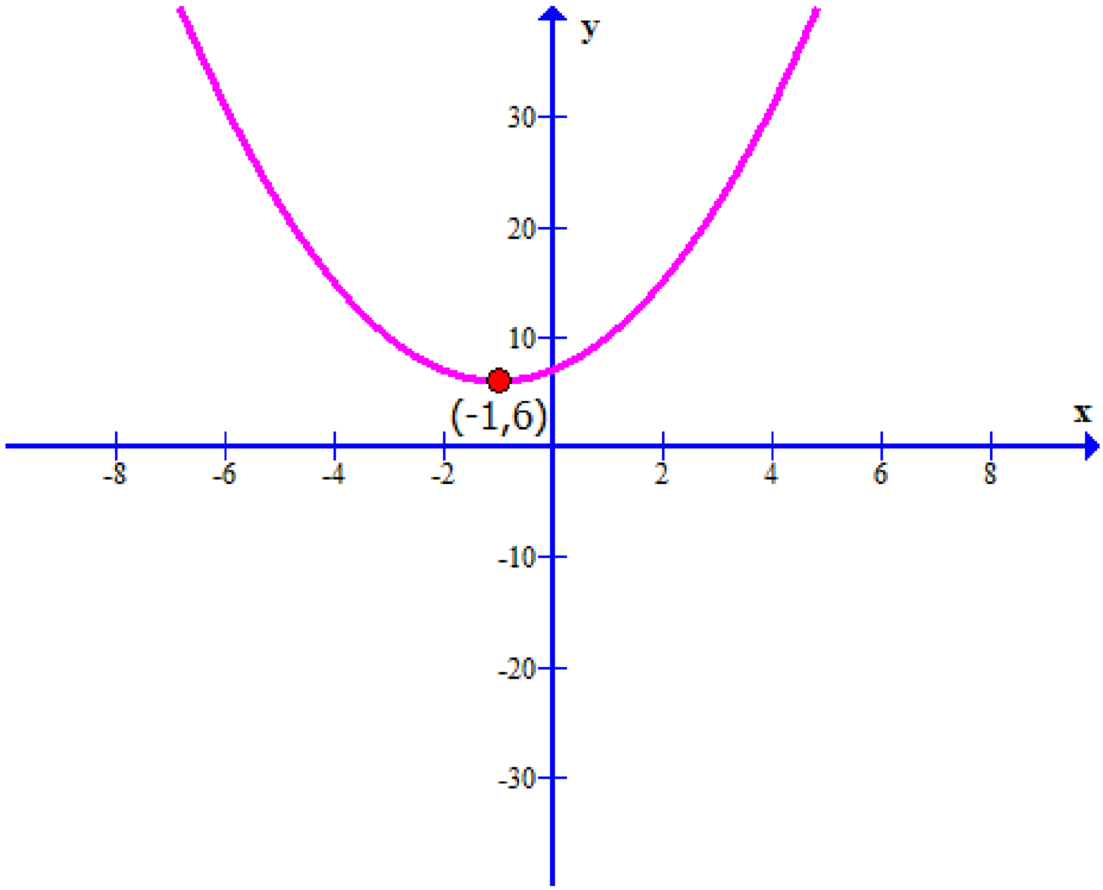
$(-1,-1) and\ (1,-1)$

f) the relative minimum value $y = -1\ when\ x = -1,1$

7)



8)



a) interval(s) where the graph is increasing. $(-1, \infty)$

b) interval(s) where the graph is decreasing. $(-\infty, -1)$

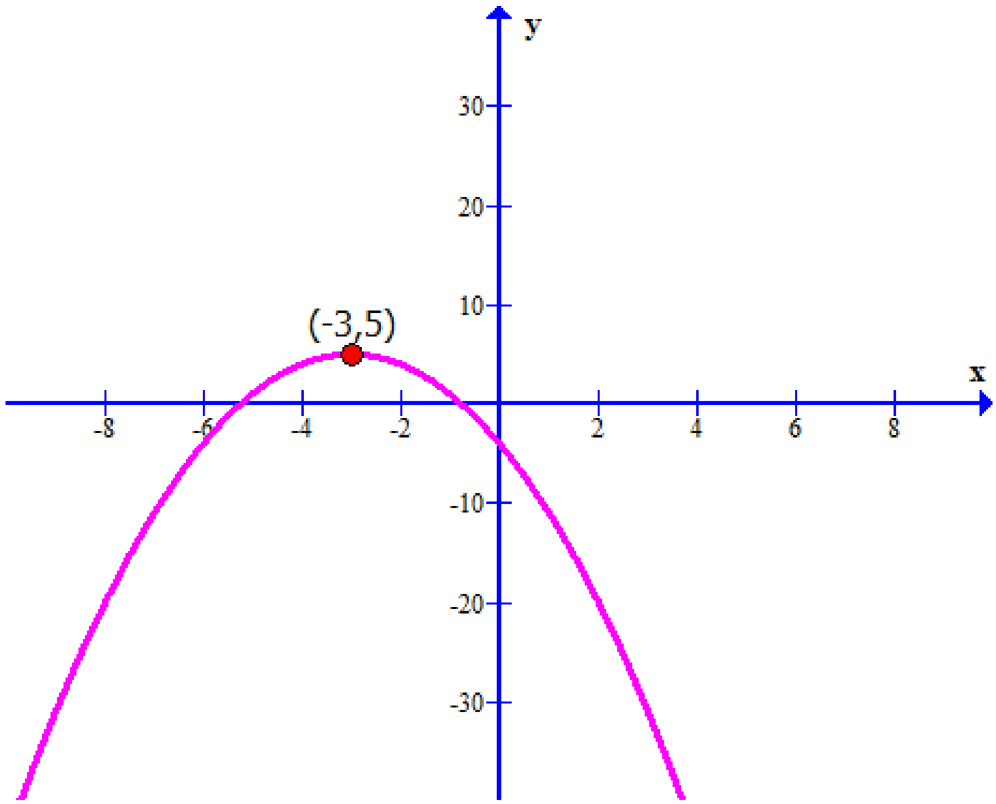
c) the coordinates of relative maximum point, if any none

d) the relative maximum value none

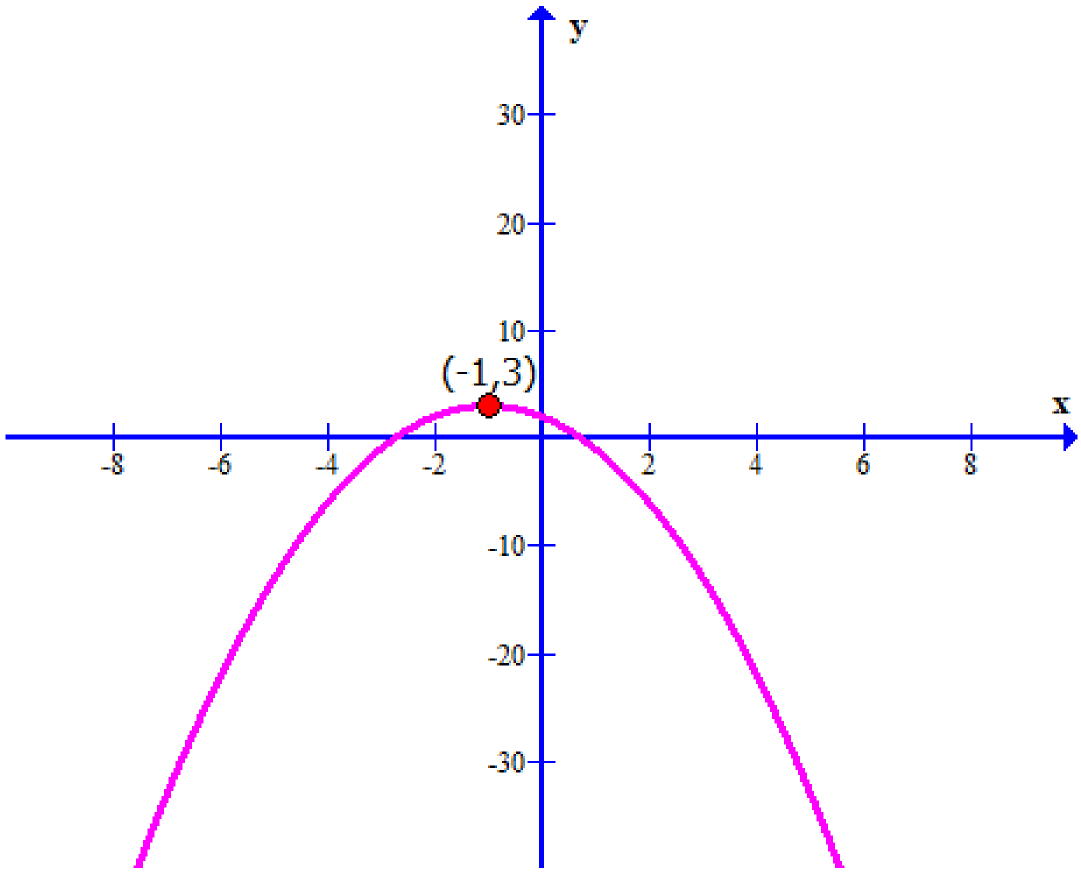
e) the coordinates of the relative minimum point if any $(-1,6)$

f) the relative minimum value $y = 6 \; when \; x = -1$

9)

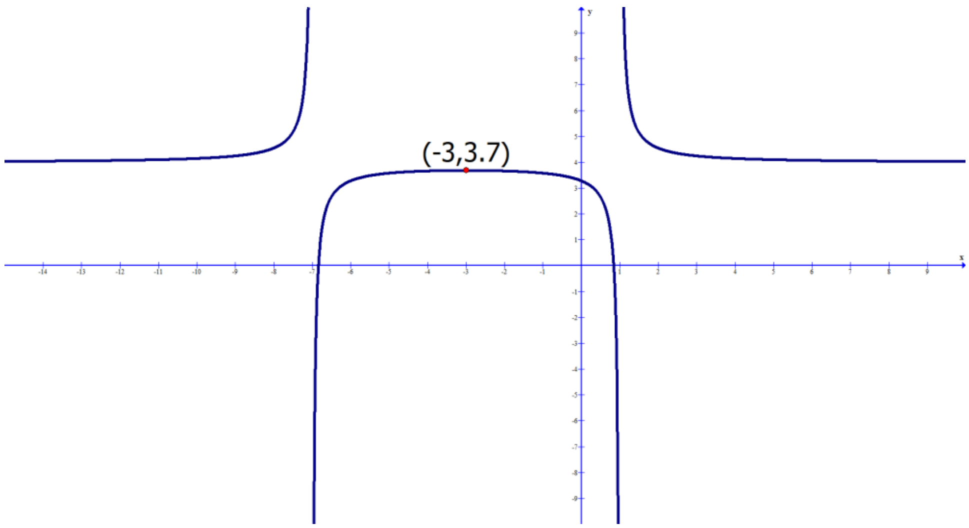


10)

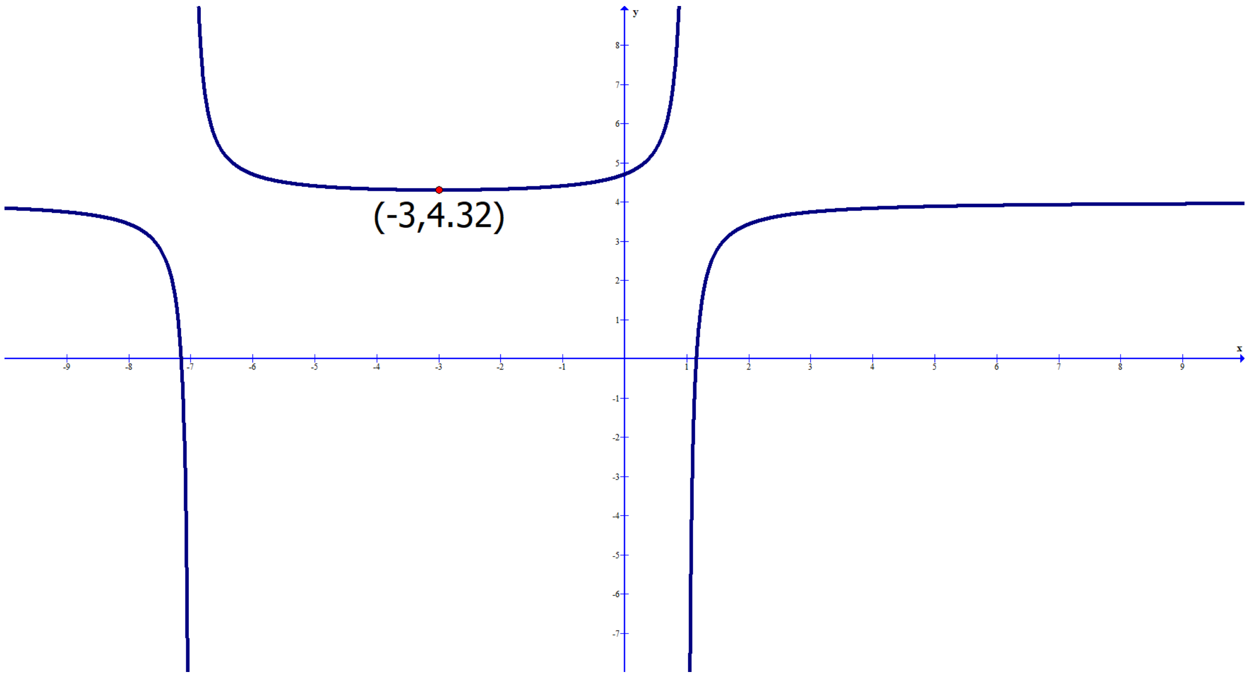


a) interval(s) where the graph is increasing. $(-\infty, -1)$
b) interval(s) where the graph is decreasing. $(-1, \infty)$
c) the coordinates of relative maximum point if any $(-1,3)$
d) the relative maximum value $y = 3 \; when \; x = \; -1$
e) the coordinates of the relative minimum point if any none
f) the relative minimum value none

11)



12)



a) interval(s) where the graph is increasing. $(-3,1) \cup (1, \infty)$
b) interval(s) where the graph is decreasing. $(-\infty, -7) \cup (-7, -3)$
c) the coordinates of relative maximum point, if any none
d) the relative maximum value none
e) the coordinates of the relative minimum point if any $(-3, 4.32)$
f) the relative minimum value $y = 4.32 \; when \; x = -3$

(Minimum Homework: 1 – 11 odds 13, 17, 21, 25)

#13 – 26: For each function find the following:

a) f'(x)

b) the critical numbers

c) interval(s) where the graph is increasing.

d) interval(s) where the graph is decreasing.

e) the coordinates of relative maximum point if any

f) the relative maximum value

g) the coordinates of the relative minimum point if any

h) the relative minimum value

13) $f(x) = x^2 - 6x + 3$

14) $f(x) = 2x^2 - 8x + 1$

a) f'(x) $f'(x) = 4x - 8$
b) the critical numbers $x = 2$
c) interval(s) where the graph is increasing. $(2, \infty)$
d) interval(s) where the graph is decreasing. $(-\infty, 2)$
e) the coordinates of relative maximum point if any none
f) the relative maximum value none
g) the coordinates of the relative minimum point if any $(2, -7)$
h) the relative minimum value $y = -7 \; when \; x = 2$

#13 – 26: For each function find the following:

a) f'(x)

b) the critical numbers

c) interval(s) where the graph is increasing.

d) interval(s) where the graph is decreasing.

e) the coordinates of relative maximum point if any

f) the relative maximum value

g) the coordinates of the relative minimum point if any

h) the relative minimum value

15) $f(x) = x^2 - 3$

16) $f(x) = x^2 + 5$

Skipping for time

#13 – 26: For each function find the following:

a) f’(x)

b) the critical numbers

c) interval(s) where the graph is increasing.

d) interval(s) where the graph is decreasing.

e) the coordinates of relative maximum point if any

f) the relative maximum value

g) the coordinates of the relative minimum point if any

h) the relative minimum value

17) $f(x) = x^3 - 12x + 4$

18) $f(x) = x^3 - 48x + 18$

a) f'(x) $f'(x) = 3x^2 - 48$
b) the critical numbers $x = 4, -4$
c) interval(s) where the graph is increasing. $(-\infty, -4) \cup (4, \infty)$
d) interval(s) where the graph is decreasing. $(-4,4)$
e) the coordinates of relative maximum point if any $(-4,146)$
f) the relative maximum value $y = 146\ when\ x = -4$
g) the coordinates of the relative minimum point if any $(4, -110)$
h) the relative minimum value $y = -110\ when\ x = 4$

#13 – 26: For each function find the following:

a) f'(x)
b) the critical numbers
c) interval(s) where the graph is increasing.
d) interval(s) where the graph is decreasing.
e) the coordinates of relative maximum point if any
f) the relative maximum value
g) the coordinates of the relative minimum point if any
h) the relative minimum value

19) $f(x) = -x^3 - 3x^2 + 45x - 5$ 20) $f(x) = -x^3 - 6x^2 + 24x - 9$

21) $f(x) = \frac{x+2}{x-5}$

22) $f(x) = \frac{x+3}{x-7}$

a) f'(x) $f'(x) = \frac{-10}{(x-7)^2}$

b) the critical numbers $x = 7$

c) interval(s) where the graph is increasing. never

d) interval(s) where the graph is decreasing. $(-\infty, 7) \cup (7, \infty)$

e) the coordinates of relative maximum point if any none

f) the relative maximum value none

g) the coordinates of the relative minimum point if any none

h) the relative minimum value none

#13 – 26: For each function find the following:

a) f'(x)

b) the critical numbers

c) interval(s) where the graph is increasing.

d) interval(s) where the graph is decreasing.

e) the coordinates of relative maximum point if any

f) the relative maximum value

g) the coordinates of the relative minimum point if any

h) the relative minimum value

23) $f(x) = \frac{x-4}{x+1}$

24) $f(x) = \frac{x-6}{x+3}$

25) $f(x) = xe^{3x}$

26) $f(x) = xe^{2x}$

a) f'(x) $f'(x) = e^{2x}(2x+1)$
b) the critical numbers $x = -1/2$
c) interval(s) where the graph is increasing. $(-\frac{1}{2}, \infty)$
d) interval(s) where the graph is decreasing. $\left(-\infty, -\frac{1}{2}\right)$
e) the coordinates of relative maximum point if any none
f) the relative maximum value none
g) the coordinates of the relative minimum point if any $\left(-\frac{1}{2}, \frac{-1}{2e}\right)$
h) the relative minimum value $y = -\frac{1}{2e}\ when\ x = -\frac{1}{2}$